Characterization of peristaltic flow during the mixing process in a model human stomach

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Characterization of peristaltic flow during the mixing process in a model human stomach

Numerical simulations are performed to investigate the characteristics of peristaltic flow in a model stomach during the mixing and digestion process. The geometrical model for the stomach consists of an axisymmetric tube of varying diameter with a wall at one end, representing the antrum and closed pylorus. The antral contraction waves (ACWs) that produce the peristaltic flow are modeled as traveling waves that deform the boundary and consequently the computational mesh. This geometrical model is implemented into the open source code OpenFOAM. A parametric study is performed in which the fluid viscosity, wave speed, wave width, and maximum relative occlusion are varied. The effect of these parameters on the retropulsive jet induced near the pylorus and the recirculation between pairs of consecutive ACWs is investigated. Both of these flow features contribute to the mixing and digestion process. The retropulsive jet is quantified by its peak velocity and length along the centerline. For each wave geometry, these quantities are found to be independent of the Reynolds number for low Reynolds numbers, while for Reynolds numbers exceeding one, the peak centerline velocity decreases and the jet length increases as the Reynolds number increases. Moreover, the velocity and pressure curves are found to scale with the wave speed at low Reynolds numbers. Between different wave geometries, scaling laws are proposed and tested for the peak centerline velocity and jet length. Particle tracking and vorticity plots show that mixing efficiency increases when the relative occlusion increases, as well as when the viscosity or wave width decreases.

I. INTRODUCTION

Peristalsis in the human body is the muscular contraction and relaxation of vessel walls that induces a wavelike motion and thus generates a flow of the material inside. Peristalsis plays a significant role in gastric digestion and mixing within the human stomach and in the transportation of material through the digestive tract and other small vessels. Peristaltic transport in the human body has been studied extensively (see, e.g., Refs. 1–7). The focus of this study is the peristaltic and mixing flow that is produced in the stomach at the beginning of the digestion process.

The human stomach is a J-shaped, muscular, hollow, and dilated part of the gastrointestinal tract, located between the esophagus and the first part of the small intestine (duodenum). Anatomically, the stomach is subdivided into the fundus, the corpus, and the antrum (see Fig. 1). The curved, twisted shape of the stomach not only supports gastric mixing, but also separates the stomach into a reservoir and mixing regions. The principle functions of the human stomach are as follows: the upper part of the stomach (the fundus and the proximal corpus) acts as a reservoir of chewed food (bolus) that enters the stomach through the esophagus via the lower esophageal sphincter, while the lower part of the stomach (the antrum and the distal corpus) is responsible for mechanical forces and fluid motions that promote not only the breakdown and mixing of gastric content, but also its chemical digestion, absorption, and transport. The mechanical forces and fluid motion are caused by peristalsis induced by antral contraction waves (ACWs), which are wavelike muscular contractions of the stomach walls. After that, the pyloric sphincter controls the passage of partially digested food from the stomach into the duodenum where peristalsis transports the material through the rest of the small intestine (see Refs. 10–15). Physiological studies of the human stomach and the gastrointestinal tract have been performed by various researchers over the past several decades, including Kelly, Urbain et al., and Pallotta.

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et al.,16 Indireshkumar et al.,17 Kwiatek et al.,18 and Imai et al.15 In particular, the study of Kwiatek et al.18 used magnetic resonance imaging (MRI) to quantify the geometry and the speed of ACW in a sample of healthy humans. The statistics from this study indicated average values and ranges for various parameters that describe ACW motility.

Developing an in vitro system capable of reproducing the fluid mechanical forces that promote digestion has been very difficult to achieve. Since the beginning of the 1990s, a series of in vitro systems have been developed to analyze human digestion, including those of Aoki et al.,19,20 Molly et al.,21 Yoo and Chen,22 Kong and Singh,23,24,25 and Wickham et al.26 Notably, the Human Gastric Simulator (HGS) of Kong and Singh24 consisted of a latex vessel with a series of motor-controlled rollers to model the stomach and ACW. Inside the vessel was a mesh bag to allow small particles (less than 2 mm) to escape from the vessel during the digestion experiments, hence mimicking the filtering role of the pylorus. Unlike previous in vitro models, the HGS included continuous peristaltic motion produced by the ACWs. This allowed it to reproduce the retropulsion, or retropulsive jet, that occurs in the stomach when the ACW approaches the pylorus and that contributes greatly to the breakdown of food particles. The authors studied the digestion and emptying process of the stomach, determining, in particular, the particle size distribution of the sample meal and the volume of the digesta inside the HGS over the digestion time of 3 h. More recently, Dufour et al.26 constructed a simplified model stomach that included a tube of a constant diameter that was closed at one end to represent the antrum and closed pylorus. The ACWs were modeled by moving a hollow piston through the tube. Unlike the previous authors, the focus was on quantifying the retropulsive jet and the details of its effect on the breakdown of food particles, represented by liquid drops of varying viscosity. By measuring the velocity field using the Ultrasonic Doppler Velocity Profile (UVP),27 the authors showed that this simplified model produced the retropulsive jet that occurs in the stomach and that their results for different fluids, relative occlusions, and wave speeds showed the same behavior as seen in other experimental and numerical studies. As the focus was on the retropulsive jet, and not the emptying process, the authors treated the pylorus as a closed impermeable surface.

Various mathematical models and numerical simulations have also been used to study the gastric mixing and digestion functions of the stomach. However, due to the biological, physiological, and neurological complexity of the gastrointestinal tract and the digestion process, developing a mathematical model of the entire stomach (e.g., fundus, corpus, antrum, and pyloric sphincter) and simulating all its gastric functions (e.g., chemical and mechanical disintegration of food material and gastric emptying) and associated neurophysiology (e.g., the opening and closing of the pyloric sphincter) remains a challenge. Pal et al.17 used statistical data reported by Pallotta et al.16 and Indireshkumar et al.17 to develop a two-dimensional planar computational model of gastric mixing and emptying of the stomach. They also used magnetic resonance imaging (MRI) movies of a healthy human stomach to capture the total propagation time and overall pattern of contraction. Later, their two-dimensional model and the lattice-Boltzmann method were used to reveal the “Magenstrasse” for gastric emptying.18 The observations of Pal et al.28 are consistent with the physiological studies of Indireshkumar et al.17 and Hausken et al.29 which suggest that gastric emptying tends to occur during the period when the ACW is distant from the pylorus and the terminal antrum has not initiated segmental contraction.

Pullan et al.30 developed an anatomical model of the stomach based on the data of the visible human project that was done by Spitzer et al.31 Subsequently, Imai et al.15 numerically investigated the flow in the stomach during gastric mixing when the pylorus was closed. They used the above anatomical model for the stomach geometry and free surface flow modeling to analyze the effects of stomach posture on gastric mixing. They found that antral recirculation transports gastric content from the distal stomach to the antrum near the pylorus, where it is then mixed by retropulsive flow. They concluded that gastric content inside the antral recirculation is well mixed independent of the initial location, whereas the content outside the recirculation region is poorly mixed. More recently, Skamniotis et al.32 used the finite element analysis to model food separation patterns and studied food bolus disintegration rates in the stomach.

Like Imai et al.,32 mentioned above, Ferrua and Singh33,34 developed a three-dimensional computational model of the entire stomach in which the pylorus was kept closed. They studied the effects of the content viscosity on gastric flow during the digestion process. They assumed that the stomach was fully filled with liquid contents, ignored the effect of gravity, and considered only the instantaneous flow and mixing. Moreover, they assumed that the stomach geometry was planar-symmetric, i.e., they assumed symmetry of the stomach with respect to the plane that bisects the stomach along its
lesser and greater curvatures. In their study, the commercial computational fluid dynamics (CFD) software package FLUENT\cite{35} was used for numerical computations, and the characteristic dimensions of the stomach geometry were obtained from Keet\cite{36} and Schulze\cite{37} in conjunction with the MRI analysis of Pal et al.\cite{38}

While the above studies investigated the effect of fluid or geometrical parameters on ACW flow in the stomach, they did not present a detailed characterization of the flow in terms of the Reynolds number over a wide range of relevant Reynolds numbers, or provide an understanding of scaling principles. Since the geometry of the stomach and ACW motility vary significantly between individuals, establishing these principles is important to understand quantitatively the effect these variations have on the mixing and digestion process. The goal of the present study is to address these issues in order to obtain a better characterization of the flow that is produced inside the human stomach during the mixing process at the beginning of digestion. As in the studies of Ferrua and Singh\cite{39,40} and Imai et al.,\cite{41} we do not consider the gastric emptying process and, therefore, assume that the pylorus remains impermeable and closed. As noted by these authors, this is a reasonable assumption at the beginning of the digestion process (e.g., first few minutes) or when the emptying rate resulting from the opening of the pylorus is small or negligible. Unlike the other studies, our focus is on the lower part of the stomach or antrum. A geometrical model for the antrum was developed, consisting of a conical-shaped tube that is closed at the narrow end, with the ACWs simulated by deforming the domain boundary by traveling waves that move toward the pylorus. This conical shape is a simplification of the geometrical models developed by Ferrua and Singh,\cite{39,40} Imai et al.,\cite{41} and Pal et al.,\cite{38,42} which represent larger portions of the stomach and take into account its J-shape. The advantages of our simplified model are that it allows for a more detailed parametric study due to the reduction in computational cost, while still capturing the relevant flow behavior we want to characterize.

Our geometrical model and boundary deformation algorithm were implemented into the open source software package, Open-FOAM,\cite{43} that was used for the flow calculations. Simulations were performed for a wide range of Newtonian fluids that reflect gastric contents, several wave speeds, and different wave widths and relative occlusions. Whenever possible, comparisons were made with experimental or other simulation data to validate our model and methods. Our model was shown to produce the two basic flow patterns known from the other studies: the retrograde jetlike motion, called the retropropulsive jet, induced by the ACWs near the closed pylorus, and the circulatory flow between the ACWs. Both motions contribute to the mixing process in the stomach and are largely regarded as the main mechanical mechanisms driving the gastric digestion. In this study, we characterize both these flow patterns in terms of the Reynolds number and propose scaling laws for the retropropulsive jet, particularly between different wave geometries (i.e., wave width and maximum relative occlusion).

II. COMPUTATIONAL MODEL

A. Geometry

We are interested in simulating ACW flow in the lower part of the human stomach, or antrum, when the pylorus is closed. As mentioned above, the pylorus may be considered closed at the beginning of digestion or when the emptying rate is relatively low or negligible. The geometrical model for the undeformed antrum was taken to be a circular tube with linearly decreasing radius, with the ACWs represented by a sequence of traveling waves that deform the boundary walls. The computational domain was an axisymmetric slice of this conical tube consisting of a 5° wedge of one cell thickness running along the axis of symmetry. In this axisymmetric slice, the ACWs moved with uniform speed along the boundary and were generated by moving the mesh points of the outer (upper) wall along the wedges.

A schematic diagram of the stomach and our computational domain for the antrum is illustrated in Fig. 1, along with the relevant dimensions and geometrical parameters. The characteristic dimensions were obtained from the studies of Kwiatek et al.\cite{44}, Keet, Schulze, Pal et al.,\cite{38,45} and Ferrua and Singh.\cite{39} Specifically, the axial length of the tube was 150 mm with diameters of 100 mm and 10 mm at its widest point and at the pyloric ring, respectively. An ACW was initiated every 20 s at the left end of the geometry, at approximately 145 mm from the pylorus. The relative occlusion of an ACW at a fixed axial location is defined by RO(x) = 1 − H(x)/H0(x), where H defines the occlusion radius and H0 defines the radius of the undeformed tube (refer Fig. 1). A relative occlusion of RO = 0 corresponds to no boundary deformation, while RO = 1 corresponds to no gap underneath an ACW. As an ACW propagates toward the pylorus, its relative occlusion increases until the wave damps out shortly before the pylorus. For notational convenience, the maximum relative occlusion before the wave damps out was denoted simply by RO, with H0 and H referring to the radii at maximum relative occlusion.

The deformation of the boundary was described by the parameter α = α(x, t), which gives the vertical displacement of points on the upper wall boundary. For a single wave, the deformation at a given axial position x and a given time t was defined by

\[ α(x, t) = Y_{tube}(x) - Y_{wave}(x, t), \quad x_1(t) \leq x \leq x_2(t), \]  

(1)

where \( Y_{tube}(x) \) is the radius of the undeformed tube and \( Y_{wave}(x, t) \) is the tube radius under the wave (that is, they are the y-coordinates of a point on the undeformed upper wall and on the wave at the same x and t), and \( x_1(t) \) and \( x_2(t) \) are the x-coordinates of the tail and leading end of the wave, respectively.

The equation of the undeformed wall was given by

\[ Y_{tube}(x) = y_{max} - (x - x_{min}) \tan θ, \]  

(2)

where \( y_{max} \) and \( y_{min} \) are the maximum and minimum y-components of the points on the undeformed upper wall, \( x_{max} \) and \( x_{min} \) are the maximum and minimum x-components defining the domain, and \( θ = \tan^{-1}[(y_{max} - y_{min})/(x_{max} - x_{min})] \) is the inclination angle of the upper wall from the x-axis. For the tube dimensions used in this study (shown in Fig. 1), \( θ = 16.7° \).

The ACWs were taken to be parabolic in shape so that \( Y_{wave}(x, t) \) was found by rotating the parabola

\[ f(x, t) = f_0(t) + A(\dot{x} - \ddot{x}_0(t))^2 \]  

(3)
clockwise by $\theta$ degrees. After rotation, the vertex of the parabola was denoted by $(x_0(t), y_0(t))$, and the parabola coefficient $A > 0$ was given by $A = 4\beta/\lambda^2$, where $\lambda$ is the wave width and $\beta$ is the wave amplitude. Although the wave amplitude may be varied during the calculation, we took it to be constant here. It may be written as $\beta = (H_0 - H) \cos \theta = H_0 \cos \theta$.

The vertex $(x_0(t), y_0(t))$ of the parabola moved with uniform speed, $c$, in the direction parallel to the upper wall according to

$$
x_0(t) = \bar{x}_0 + ct \cos \theta, \quad y_0(t) = \bar{y}_0 + ct \sin \theta,
$$

where $(\bar{x}_0, \bar{y}_0)$ is its initial location at time $t = 0$. In a similar manner, the $x$-coordinate of the two ends of the wave are given by

$$
x_i(t) = \bar{x}_i + ct \cos \theta; \quad i = 1, 2,
$$

where $\bar{x}_i$’s are their initial values at time $t = 0$. Between any two consecutive ACWs and for the values of $x$ outside the interval $x_1(t) \leq x \leq x_2(t)$, there was no boundary deformation and $a(x, t) = 0$. Note that the dimensions of the domain $(x_{\text{min}}, x_{\text{max}}, y_{\text{min}}, y_{\text{max}})$, the wave parameters $(x_0, y_0, \lambda, \beta, \rho)$, and the wave speed ($c$) are model parameters from which all other parameters can be determined.

B. Governing equations and numerical methods

The flow is governed by the mass and momentum conservation equations for an incompressible, isothermal, and inelastic fluid

$$
\nabla \cdot \mathbf{u} = 0,
$$

$$
\rho \left( \frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} \right) = -\nabla p + \nabla \cdot \mathbf{r},
$$

where $\mathbf{u}$ is the fluid velocity, $p$ is the pressure, $\rho$ is the density, and $\mathbf{r}$ is the viscous stress tensor. The fluids were taken to be Newtonian so that $\mathbf{r} = \mu \nabla \mathbf{u}$, where $\mu$ is the constant dynamic viscosity and $\nabla \cdot \mathbf{u}$ is the rate-of-strain tensor.

As the parabola deforms, the internal mesh must be adjusted. This mesh motion was governed by the Laplace equation

$$
\nabla \cdot (\xi \nabla v) = 0,
$$

where $v$ is the cell velocity in a given time step and $\xi$ is the preset variable of diffusivity that describes how points should be moved when solving the cell motion equation in each time step. In the model used for the directional diffusivity, the deformation of a cell is proportional to the direction of motion. Specifically, the diffusivity on a cell face $f$ with a unit normal $\mathbf{n}_f$ is given by

$$
\xi_f = \mathbf{n}_f \cdot \mathbf{D} \cdot \mathbf{n}_f,
$$

where $\mathbf{D}$ is a diagonal matrix whose diagonal entries are user-specified values to control the quality of the mesh.

Initially, the fluid velocity and pressure fields were zero. The boundary conditions for fluid velocity and pressure were as follows: At the left boundary, which represents the opening into the antrum, the velocity gradient in the normal direction and the total pressure $p_0 = p + (\rho/2)|\mathbf{u}|^2$ were set to zero. These boundary conditions allow for inflow and outflow of fluid across this boundary of the computational domain. The right boundary represents the closed pylorus, which is assumed to be impermeable and motionless. On this boundary, the velocity components were set to zero (no-slip) as was the pressure gradient in the normal direction. The upper wall was also assumed to be an impermeable and no-slip boundary. The time-dependent velocity on this boundary was determined by Eqs. (1)–(5), which give the positions of the boundary points as a function of time. Specifically, the velocity on the upper wall was the normal component of $\mathbf{u}(\mathbf{x}, t) = \Delta x(t)/\Delta t$, where $\mathbf{x}$ represents coordinates of points on the upper wall. On this boundary, the normal gradient of the pressure was set to zero. Axisymmetric boundary conditions were used along the centerline. Finally, the cell velocity, $\mathbf{v}$, was zero on the centerline, the left boundary, and the right boundary. This means that the ACWs cannot intersect the antrum inlet or the pylorus. The cell velocity on the upper wall was determined directly from the change of the boundary point positions given in Eqs. (1)–(5).

The above equations for $\mathbf{u}$, $p$, and $\mathbf{v}$ were solved using finite volume and dynamic meshing techniques within the OpenFOAM environment. (For details refer Jasak and Tukovic and Holzmann.) For this study, a C++ library was developed to implement the boundary deformation described in Eqs. (1)–(5). This library was then coupled to an OpenFOAM transient solver for a dynamically varying mesh. The flow solver uses a segregated pressure-velocity coupling algorithm in which the discrete equations for each variable are solved iteratively in each time step until the convergence is reached. Spatial discretization was achieved using standard second-order finite volume schemes, including a second-order upwind scheme for the convection terms when needed, and an implicit bounded first-order Euler method was used to discretize the temporal terms. Adaptive time steps were used to ensure a maximum Courant number of no more than 0.5.

It is important to emphasize the assumptions and limitations of our simplified model and simulations. In addition to simplifying the geometry of the antrum, we have also neglected gravity, stomach posture, surface properties of the stomach walls, and slip conditions, which may be present. Since we assumed that the pylorus remains closed and impermeable, our simulations addressed only the beginning of the digestion process (first few minutes). We did not simulate the emptying process or the extended digestion process that may last for several hours.

C. Computational details and mesh independence

Five Newtonian fluids were considered, whose viscosities ranged from $\mu = 10^{-3}$ Pa s (water) to $\mu = 10$ Pa s (honey) in factors of 10. The fluid parameters are shown in Table I. Three wave speeds

<table>
<thead>
<tr>
<th>Fluid</th>
<th>Viscosity $\mu$ (Pa s)</th>
<th>Density $\rho$ (kg/m$^3$)</th>
<th>Reynolds numbers</th>
</tr>
</thead>
<tbody>
<tr>
<td>N1</td>
<td>water</td>
<td>1000</td>
<td>RO = 0.52</td>
</tr>
<tr>
<td>N2</td>
<td>0.01</td>
<td>1000</td>
<td>3.64 9.23</td>
</tr>
<tr>
<td>N3</td>
<td>0.1452</td>
<td>1000</td>
<td>0.251 0.636</td>
</tr>
<tr>
<td>N4</td>
<td>1</td>
<td>1360</td>
<td>0.049 0.126</td>
</tr>
<tr>
<td>N5</td>
<td>honey</td>
<td>10</td>
<td>0.0049 0.0126</td>
</tr>
</tbody>
</table>
of $c = 1.15 \text{ mm/s}$, $2.3 \text{ mm/s}$, and $4.6 \text{ mm/s}$ were used, along with two maximum relative occlusions of $RO = 0.52$ and 0.80. In each case, the maximum relative occlusion was reached when the wave vertex was approximately 9 mm from the closed pylorus. Moreover, the width of the waves along the deforming wall was taken to be either 7 mm (narrow wave) or 16.5 or 18.5 mm (wide wave). Table II summarizes the simulation conditions. All values lie within the range of values reported in the literature (e.g., Ferrua and Singh$^1$ and Kwiatek et al.$^1$).

The Reynolds numbers for the fluids at the maximum relative occlusion are listed in Table I for the wave speed $c = 2.3 \text{ mm/s}$. They were computed from $Re = \rho U H / \mu$, where $\mu$ and $\rho$ are the fluid viscosity and density, $U = c (H_0 / H)^2 = c / (1 - RO)^2$ is the characteristic velocity with $H$ as the gap radius and $H_0$ as the radius of the undeformed tube at maximum relative occlusion $RO$. The corresponding Reynolds number at $c = 1.15 \text{ mm/s}$ and $c = 4.6 \text{ mm/s}$ are one-half or twice these values, respectively.

To ensure that the computational mesh was sufficiently refined, a mesh independence study was performed. Table III summarizes three meshes that were used, where the number of cells in the x- and y-direction between consecutive meshes differ by a factor of 1.5. Before boundary deformation, each mesh had a uniform distribution of cells in the axial and radial directions (x- and y-direction, respectively). Due to the decreasing tube radius, this lead to cell sizes that decreased as $x$ increased, with the smallest cells being near the pylorus. As the ACWs deformed the boundary, the sizes of the cells under the waves further decreased.

Representative results of the mesh refinement study are shown in Fig. 2, which gives the velocity profiles along the centerline when an ACW reaches maximum relative occlusion close to the pylorus. Figure 2(a) corresponds to fluid $N_1$ with the fastest wave speed and the narrow wave at $RO = 0.80$ (wave geometry 1). Figure 2(b) corresponds to the least viscous fluid $N_1$ with the middle wave speed and the wide wave at $RO = 0.80$ (wave geometry 2). The vertical dashed lines in each graph represent the ends of an ACW. This figure shows good mesh convergence with the results on the middle mesh (Mesh 2) and finest mesh (Mesh 3) being essentially mesh independent. Therefore, Mesh 2 was used in the remaining simulations.

### TABLE II: Simulation conditions used in this study. The wave coefficient $A$ is from Eq. (3) and is given by $A = 4 \beta \lambda^2$, where $\beta = (H_0 - H) \cos \theta = H_0 R \cos \theta$ is the wave amplitude. The scaling factors $\phi^{-1}$ and $\psi$ are discussed in Sec. III A. The numbers in bold indicate our standard simulation case.

<table>
<thead>
<tr>
<th>Wave geometry</th>
<th>Wave Width</th>
<th>Coefficient</th>
<th>Scaling factor $\phi^{-1}$</th>
<th>Scaling factor $\psi$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wave geometry 1</td>
<td>0.80</td>
<td>7 (narrow)</td>
<td>0.4566</td>
<td>1.643</td>
</tr>
<tr>
<td>Wave geometry 2</td>
<td>0.80</td>
<td>18.5 (wide)</td>
<td>0.0671</td>
<td>1.268</td>
</tr>
<tr>
<td>Wave geometry 3</td>
<td>0.52</td>
<td>7 (narrow)</td>
<td>0.2999</td>
<td>1.395</td>
</tr>
<tr>
<td>Wave geometry 4</td>
<td>0.52</td>
<td>16.3 (wide)</td>
<td>0.05546</td>
<td>1.110</td>
</tr>
</tbody>
</table>

### TABLE III: Computational meshes used in the axisymmetric conical simulations. Each mesh is uniform in the x-direction and nonuniform in the y-direction with the smallest cells near the pylorus boundary. The cell heights given are those near the pylorus boundary before deformation, where $R = y_{max} = 5 \text{ mm}$.

<table>
<thead>
<tr>
<th>Mesh</th>
<th>Cell distribution</th>
<th>Number of cells</th>
<th>$\Delta x/R$</th>
<th>$\Delta y/R$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mesh 1</td>
<td>$180 \times 12 \times 1$</td>
<td>2160</td>
<td>0.167</td>
<td>0.083</td>
</tr>
<tr>
<td>Mesh 2</td>
<td>$270 \times 18 \times 1$</td>
<td>4860</td>
<td>0.111</td>
<td>0.056</td>
</tr>
<tr>
<td>Mesh 3</td>
<td>$405 \times 27 \times 1$</td>
<td>10935</td>
<td>0.074</td>
<td>0.037</td>
</tr>
</tbody>
</table>

- **D. Model validation**

The simplified stomach model described above was validated with the results of Ferrua and Singh,$^{13}$ using their set of parameter values. Ferrua and Singh utilized a more complex and computationally expensive three-dimensional, planar-symmetric model that includes the curvature of the stomach walls depicted in Fig. 1. The ACWs were initiated every 20 s at a distance of 150 mm from the pylorus and had an average speed of 2.3 mm/s. This average speed was obtained as the mean of the ACW speeds along the larger and smaller curvature of their model stomach in the symmetry plane. The maximum relative occlusion of 0.80 was attained approximately 5.4 mm from the pylorus. They considered two Newtonian fluids both with a density of $\rho = 1000 \text{ kg/m}^3$ and dynamic viscosities of $\mu = 0.001 \text{ Pa s}$ and 1 Pa s. The quantities we used for comparison were the maximal velocity magnitude on the centerline near the pylorus, $|u|_{max}$, and an averaged vorticity magnitude, $|\omega_{avg}|_{max}$, where the average was taken over the region between two adjacent ACWs. These quantities were obtained at maximum relative occlusion, when the retropulsive jet was strongest.

The comparison between the two geometrical models is shown in Table IV. The maximum velocity magnitudes lie within one percent for the less viscous fluid, and within six percent for the more viscous fluid. The corresponding averaged vorticities are approximately four percent and nine percent for the low and high viscous fluid, respectively. Given the difference between the two geometries, and the fact that the wave speed of 2.3 mm/s is an averaged value in the Ferrua and Singh simulations, this agreement is very good.

Although the discrepancy increased for the higher viscous fluid, we do not expect the agreement between our results and those from the three-dimensional model to worsen for fluids with viscosity greater than 1 Pa s. This is because the Reynolds numbers for fluids with viscosity greater than or equal to 1 Pa s satisfied $Re < 0.25$.
(Stokes flow conditions), and so we would expect each model to produce results for the 10 Pa s fluid that are similar to those produced for the 1 Pa s fluid. Moreover, the discrepancy can be considered particularly acceptable in light of the savings in geometrical complexity and computational time associated with our simplified model; the three-dimensional simulations of Ferrua and Singh employed 767 800 cells, compared to 5000–11 000 cells in our conical model (depending on the mesh used). Finally, we note since Ferrua and Singh considered one ACW geometry and one ACW speed, it is not clear how sensitive their results would be to small variations in the constant average values they took for these parameters, or how much of the discrepancy we see in the predictions could be due to small perturbations.

### III. RESULTS AND DISCUSSION

The parameter values used in the model validation above are typical values for the human stomach, but these values vary. In this section, we discuss ACW flow for variations of these parameters and for a larger set of fluids. The effect of fluid viscosity, the ACW speed and width, and the maximum relative occlusion were investigated. The specific values of these parameters were given in Sec. II C. Whenever available, we make comparisons with results reported in the literature.

There are two main features of ACW flow that contribute to the mixing and mechanical disintegration of food particles. The first is a reverse jetlike pulse, called the retropulsive jet, which occurs in the most occluded region near the pylorus. The second is the recirculating flow that develops between and under the ACWs away from the pylorus. These two flow patterns are illustrated in Fig. 3, which shows the velocity magnitude field for the least and most viscous fluid ($N_1$ and $N_5$, respectively) for the narrow ACW and high RO.

These plots are at the time when the maximum relative occlusion of 0.80 has been reached and also show the streamlines at this given time instant. These types of flow patterns were also observed experimentally by Keinke et al., Schulze–Delrieu and Brown, Brasseur et al., Pallotta et al., and Boulby et al. as well as numerically by Pal et al. and Ferrua and Singh, and Imai et al. Each flow pattern is discussed in more detail in Subsections III A and III B.

### A. Retropulsive jet

In general, a characteristic of peristaltic flow in tubes is the backflow that develops underneath the wave. This is illustrated in Fig. 4, which shows an enlargement of the velocity vector field for the narrow ACWs at the maximum occlusion of 0.80. These plots are at the time when the maximum relative occlusion of 0.80 has been reached and also show the streamlines at this given time instant. These types of flow patterns were also observed experimentally by Keinke et al. Schulze–Delrieu and Brown, Brasseur et al., Pallotta et al., and Boulby et al. as well as numerically by Pal et al., Ferrua and Singh, and Imai et al. Each flow pattern is discussed in more detail in Subsections III A and III B.

| Viscosity | $u_{\text{max}}$ (mm/s) | $|\omega_{\text{avg}}|_{\text{max}}$ (s$^{-1}$) |
|-----------|-------------------------|-------------------------|
| $10^{-3}$ Pa s | 77.4 | 0.96 |
| 1 Pa s | 112.3 | 0.23 |

| Viscosity | $u_{\text{max}}$ (mm/s) | $|\omega_{\text{avg}}|_{\text{max}}$ (s$^{-1}$) |
|-----------|-------------------------|-------------------------|
| $10^{-3}$ Pa s | 78 | 1 |
| 1 Pa s | 119 | 0.21 |

### TABLE IV. Model validation by comparing the velocity and vorticity fields in our simulations with the ones reported by Ferrua and Singh. The ACWs speed and maximum relative occlusion are 2.3 mm/s and 0.80, respectively. A low and high viscosity fluid is considered, both with density $\rho = 1000$ kg/m$^3$. The parameter values used in the model validation above are typical values for the human stomach, but these values vary. In this section, we discuss ACW flow for variations of these parameters and for a larger set of fluids. The effect of fluid viscosity, the ACW speed and width, and the maximum relative occlusion were investigated. The specific values of these parameters were given in Sec. II C. Whenever available, we make comparisons with results reported in the literature.

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shows that the scaled pressures coincide, while the scaled pressure difference under the wave increases for the low viscous fluids as \( \text{Re} > 1 \) increases.

The development over time of the retropulsive jets seen in Fig. 3 is depicted in Fig. 6. The velocity magnitude fields for fluids \( N_1 \) (left column) and \( N_5 \) (right column) are plotted in time intervals of 1 s, from top to bottom. The bottom plot in each column corresponds to the maximum relative occlusion of 0.80 (note that Fig. 6 uses a broader color scale than Fig. 3). The maximum velocity magnitude for \( N_1 \) and \( N_5 \) are 63 mm/s and 94.3 mm/s, respectively. This figure shows qualitatively different retropulsive jet behavior for the low and high viscous fluids. This can be explained in terms of the Reynolds number. For high viscous fluids such as \( N_5 \), where \( \text{Re} < 1 \), there is little fluid motion outside the vicinity of the most occluded ACW. Specifically, as an ACW moves along the boundary, the backflow that is produced under the wave quickly damps out after the wave has passed due to the dominance of viscous forces over inertial forces. Therefore, when an ACW reaches maximum relative occlusion, the retropulsive jet is confined to being close to that wave. For lower viscous fluids such as \( N_1 \), where \( \text{Re} > 1 \), the backflow takes longer to damp out as the wave passes along the boundary, due to the dominance of inertial forces over viscous forces. Therefore, the retropulsive jet extends behind the wave for these fluids.

The strength of the retropulsive jet at a particular time can be quantified by its spatial extension, or length \( L \), and its maximum axial speed along the centerline, \( u_{\text{max}} = \max |u_x(x,0)| \). We characterized the length of the jet by the width of the centerline velocity curve at half the maximum axial speed. The centerline velocity for all five fluids is shown in Fig. 7(a) for the narrow ACW and high RO. In each case, the peak centerline velocity is reached close to the vertex at \( x = x_0 \). Since the axial position of the wave vertex \( x_0 \) is closer to the axial position of the tail end of the wave \( x_1 \) than to the leading end of the wave \( x_2 \), this peak velocity occurs closer to the tail end of the wave (cf. Figs. 1 and 6).

From the curves in Fig. 7(a), we see that the lowest viscous fluid \( N_1 \) has a smaller peak centerline speed than the higher viscous fluids. This illustrates that high viscous fluids experience locally larger strain rates and stresses than low viscous ones, which is essential in
the breakdown of more viscous food. This observation is supported by the corresponding centerline strain rates $\dot{\gamma} = \sqrt{(1/2)(\dot{\gamma} : \dot{\gamma})} = \sqrt{(1/2)\sum_{ij}\dot{\gamma}_{ij}\dot{\gamma}_{ji}}$ shown in Fig. 7(b). Although the strain rates and stresses for the low viscous fluid are smaller, the retropulsive jet acts over a larger region ($L = 15.9$ and $3.3$ mm for the lowest and highest viscous fluids, respectively). Therefore, fluid breakdown is reduced for the low viscous fluid but the distribution of fluid particles is enhanced. This results in an improved mixing for less viscous materials. This is discussed further in Sec. III B. These observations are in agreement with those obtained with the more complex three-dimensional model of Ferrua and Singh.33
Note that the centerline strain rates shown in Fig. 7(b) exhibit two positive peaks, one related to stretching (right peak) and one related to compression (left peak). The values are positive because the strain rate was obtained from the magnitude of the rate of strain tensor. The trough between the peaks is located near the axial position of the wave vertex. As a food particle moves under the ACW and away from the pylorus, it first experiences stretching due to fluid acceleration over the leading part of the wave, and then experiences compression due to the deceleration over the tail part of the wave. The figure shows that compression (left peak) is somewhat stronger than stretching for the most viscous fluids (Re < 1), but that stretching (right peak) becomes increasing more dominant for low viscous fluids (Re > 1).

The length $L$ of the retropulsive jet and the peak centerline velocity are shown in Fig. 8 as a function of the Reynolds number for all fluids and wave geometries. Log-log scales were used to distinguish better between the different curves, particularly the peak centerline velocity curves. In all cases, we see that the strength of the retropulsive jet is independent of viscosity when $Re < 1$ (i.e., for more viscous fluids), whereas for larger Reynolds numbers, the jet length increases and the peak centerline velocity decreases as viscosity decreases (i.e., as Re increases). In the latter case (i.e., when Re > 1), the viscosity is seen to have a larger influence on the jet length than on the peak centerline velocity. This behavior agrees with the experiments of Dufour [26] who found that as the viscosity decreased, with the Reynolds number increasing from $Re < 1$ to $Re \gg 1$, the peak centerline velocity decreased and the length of the retropulsive jet increased.

1. Effect of wave speed

The effect of wave speed was determined by halving and doubling the standard wave speed of $c = 2.3$ mm/s. The simulations were performed for the lowest viscous fluid (N₁) and a high viscous fluid (N₃). The corresponding Reynolds numbers in Table I are accordingly decreased and increased by a factor of two for $c = 1.15$ mm/s and $c = 4.6$ mm/s, respectively. For the low viscous fluid N₁, the Reynolds numbers are in the range $18 \leq Re \leq 185$, and for more viscous fluid N₃, the Reynolds numbers remain small, $0.125 \leq Re \leq 1.27$.

The centerline axial velocity and kinematic pressure for all wave geometries are shown in Fig. 9 for fluid N₃ ($0.125 \leq Re \leq 1.27$). The velocity curves have been scaled by the wave speed $c$, and the pressure curves have been scaled by $P = v c / H$, where $v = \mu / \rho$ is the kinematic viscosity. The solid curves correspond to the wave speed of $c = 2.3$ mm/s, while the open and solid symbols correspond to wave speeds of $c = 1.15$ mm/s and 4.60 mm/s, respectively. For each relative occlusion and wave width, the velocity curves coincide, showing that the centerline velocities near the pylorus scale with the wave speed for fluid N₃. The scaled pressure curves also coincide for a given wave geometry. Note that since $H$ is the same for a given wave geometry, this figure indicates that pressure scales with wave speed. That is, when the wave speed doubles, both the centerline velocity and centerline pressure double.

For the low viscous fluid N₁, where Reynolds numbers are large, this scaling is not applicable. Figure 10(a) shows the scaled
centerline velocity curves for RO = 0.80. Although the whole curves do not scale, the peak velocity value did increase by a constant factor of about 1.8 when the wave speed was doubled. That is, the peak centerline velocity scaled with 0.9c. The same was true for RO = 0.52 (not shown). This observation was also made in the experiments of Dufour et al.\textsuperscript{26} who used a tube of constant radius to model the antrum. Specifically, for a low viscous fluid with \( \mu = 3.2 \) mPa s, the peak centerline velocity was found to increase by a factor of approximately 2.5 when the wave speed tripled from 2.5 mm/s to 7.5 mm/s. In this case, the peak centerline velocity scaled with 0.83c.

The different velocity scaling for the low and high viscous fluids is also reflected in the different axial velocity profiles in cross sections perpendicular to the centerline. This is shown in Fig. 10(b) for the narrow wave at RO = 0.80. This figure shows axial velocity profiles in cross section taken at the wave vertex, where the maximum relative occlusion of 0.80 is reached. The cross-sectional velocity for the high viscous N\textsubscript{3} fluid (solid symbols) scales with wave speed, but does not scale for the low viscous N\textsubscript{1} fluid (open symbols). The solid curve through the scaled N\textsubscript{3} data represents the parabolic velocity profile for fully developed Poiseuille flow at the given centerline velocity. The fact that this curve agrees very well with the data points indicates that the local flow field under the wave acts like pressure-driven Poiseuille flow for low Reynolds numbers. However, at high Reynolds numbers, the flow under the wave cannot fully develop, and the velocity profile looks more uniform (i.e., flatter). The flatness of the profile increases with Reynolds number (i.e., wave speed) due to the longer time that is required for pressure-driven flow to develop at higher Reynolds numbers.

As with the centerline velocity, the pressure curves for large Reynolds numbers did not scale the same way they did for the low Reynolds numbers. Instead, the pressure difference along the centerline was found to increase by factors in the range of 2.6–3.1 as the wave speed doubled. These values come closer to the value predicted by Bernoulli’s equation than to the value predicted by the Hagen-Poiseuille equation. According to the Bernoulli equation, the pressure difference along a streamline for inviscid fluids scales with the square of the velocity along the streamline. Therefore, if the velocity along the centerline increases by a factor of 1.8 as the wave speed doubles, the pressure difference along the centerline would increase by a factor of 1.8\(^2\) = 3.24 according to Bernoulli’s equation, compared to 1.8 predicted by the Hagen-Poiseuille equation.

The dimensionless retropulsive jet length and the peak centerline velocity for all five fluids and three wave speeds are plotted in Fig. 11 as a function of the Reynolds number. The velocity was nondimensionalized by the wave speed c and the jet length was nondimensionalized by the gap radius \( H \) at maximum relative occlusion. The open symbols represent the data in the log-log plots of Fig. 8 for fluids N\textsubscript{1}–N\textsubscript{5} at the wave speed of \( c = 2.3 \) mm/s. The data for the different wave speeds for fluids N\textsubscript{1} and N\textsubscript{3} are overlayed and indicated by solid symbols. On each curve, the three right-most solid symbols correspond to N\textsubscript{1}, and the three left-most solid symbols correspond to N\textsubscript{3}. We see that for a given wave geometry, all data lie on the same curve.
2. Effect of wave geometry

Figure 11 also shows the effect of the wave width and maximum relative occlusion on the strength of the retropulsive jet. Along with Fig. 8, this figure shows that increasing the wave width generally causes the length of the retropulsive jet to increase (particularly for \( \text{Re} < 1 \)) and the peak centerline velocity to decrease. Conversely, as the maximum relative occlusion increases, the peak centerline velocity increases and the retropulsive jet length generally decreases.

For the three most viscous fluids (where \( \text{Re} < 1 \)), the wave width has a larger influence on the length of the retropulsive jet than the relative occlusion does, while the relative occlusion has a larger influence on the peak centerline velocity. Specifically, for \( \text{RO} = 0.80 \), the retropulsive jet length for the wide wave width is about 2.2 times larger than for the narrow wave, while the peak centerline velocity is only 1.3 times lower (for \( \text{RO} = 0.52 \), these numbers are reduced to 1.6 and 1.1, respectively). For the narrow wave width, the peak centerline velocity for \( \text{RO} = 0.80 \) is 7.2 times higher than for \( \text{RO} = 0.52 \), while the retropulsive jet length is only 1.8 times lower (for the wide wave width, these numbers are reduced to 6.4 and 1.3, respectively).

For \( \text{Re} > 1 \), the effect of the wave width and relative occlusion on the retropulsive jet length and the peak centerline velocity is diminished as the Reynolds number is increased (as the viscosity decreases). For the least viscous fluid (\( \text{N}_1 \)), the retropulsive jet length is nearly the same for all wave widths (as indicated in Fig. 11 by the three rightmost solid symbols on each curve) and both maximum relative occlusions (as indicated in Fig. 8 by the rightmost point on each curve), while the peak centerline velocity is nearly independent of the wave width but still depends strongly on relative occlusion. As the relative occlusion increases, the peak centerline velocity for the low viscous fluids increases by nearly the same factors as for the high viscous fluids, namely, 6.7 for the narrow wave (compared to 7.2 for the high viscous fluids) and 6.2 for the wide wave (compared to 6.4 for the high viscous fluids).

The fact that the high viscous fluids (\( \text{Re} < 1 \)) are affected more by the wave geometry than the low viscous ones (\( \text{Re} > 1 \)) is due to the confinement of the retropulsive jet of the high viscous fluids close to the ACW at maximum relative occlusion [see Fig. 7(a) and the discussion at the beginning of this section]. In contrast, the retropulsive jet for the low viscous fluids extends far behind the leading wave. Therefore, low viscous fluids should not be appreciably affected by the wave width, but the peak centerline velocity should still increase with maximum relative occlusion due to the decrease in cross-sectional area of the tube.

3. Retropulsive jet scaling between wave geometries

Part of the change in velocity with wave geometry is due to the change in the cross-sectional area under the wave. Figure 12(a) shows the peak centerline velocity nondimensionalized by the characteristic velocity \( U = c(H_0/H)^2 = c(1/(1 - \text{RO}))^2 \), which takes into account the change in cross-sectional area at the wave vertex. Using this scaling, we see that the narrow waves result in a larger peak centerline velocity.

The remaining change in velocity with wave geometry is due to the difference in velocity boundary values on the wave. This difference is due to the different distances that boundary points on the wave must move in a fixed amount of time. At a given wave width, the velocity boundary values will be larger for the larger relative occlusion since the boundary points must move farther down. At a given relative occlusion, the velocity boundary values will be larger for the narrow wave since the boundary of the wave is steeper.

As a consequence, the volumetric flow rate \( Q \) of the retropulsive jet (measured at maximum occlusion) increases as either the relative occlusion increases or the wave width decreases. For a wave speed of \( c = 2.3 \text{ mm/s} \) and \( \text{RO} = 0.80 \), \( Q = 342.9 \text{ mm}^3/\text{s} \) and \( 264.5 \text{ mm}^3/\text{s} \) for the narrow and wide wave, respectively. For \( \text{RO} = 0.52 \), these flow rates decrease to \( Q = 268.8 \text{ mm}^3/\text{s} \) and \( 234.2 \text{ mm}^3/\text{s} \) for the narrow and wide wave, respectively. These variations are up to 46%. If these variations in volumetric flow rate are accounted for, the peak centerline velocity for all four wave geometries coincide for \( \text{Re} < 1 \). This is shown in Fig. 12(b), where the data is scaled to the case of the narrow wave at \( \text{RO} = 0.80 \) (wave geometry 1) by multiplying each curve by the ratio \( Q_{k1}/Q_{k} \), where \( Q_{k} \) is the volumetric flow rate in wave geometry \( k \). Note that this adjustment of the dimensionless velocity \( u_{\text{max}}/U \), where \( U = c(H_0/H)^2 \), to account for variations in the volumetric flow rate has the effect of scaling the velocity \( u_{\text{c}} \) by the average jet velocity \( V_{\text{avg}} = Q/(\pi H^2) \).

The above values of the volumetric flow rate were computed from the respective velocity fields at maximum relative occlusion. Ideally, a scaling factor for \( u_{\text{max}}/U \) and \( Q \) at low Reynolds numbers can be found \textit{a priori} without the need to calculate the velocity.
field. We propose a factor here based on empiricism. First, note that the velocity on the wave, and hence, the centerline velocity and the volumetric flow rate, is related to the coefficient $A$ in the parabola describing the wave shape, Eq. (3), which is given by $A = (4H_0 \cos \theta)/\lambda^2$, where $\theta = 16.7^\circ$ is the angle of the conical-shaped antrum. Therefore, at maximum relative occlusion, $A$ follows the same trend as $Q$ and $u_{\max}/U$, that is, it increases when RO increases or when $\lambda^2$ decreases (the values of $A$ for the four wave geometries are given in Table II). Second, the volumetric flow rate and the peak centerline velocity are determined by the imposed velocity on the leading portion of the wave only, that is, on the part of the wave where the $y$-component of imposed velocity is negative (i.e., fluid is being pushed downward). This occurs between the point $P_m(x_m, y_m)$ where the minimum gap width is attained and the rightmost point $P_2(x_2, y_2)$ on the wave. Since the velocity boundary condition is applied in the normal direction to the wave, the velocity on the wave boundary depends on the normal to the wave boundary. This normal direction varies over the boundary between points $P_m$ and $P_2$. A simple measure for the normal direction is the slope $s$ of the normal line to the secant between points $P_m$ and $P_2$.

Taking into account the above observations, a proposed scaling factor $\phi$ for $u_{\max}/U$ and $Q$ is given by

$$\phi^{-1} = \sqrt{AH_m|s| \lambda} = \frac{2\sqrt{H_m H_0 \cos \theta}}{\lambda} |s| \lambda^{3/2},$$

where the slope of the normal line is given by $|s| = (x_2 - x_m)/(y_2 - y_m)$ and $H_m = H_0 - (x_m - x_0) \tan \theta$ is the radius of the undeformed tube at $x = x_m$ (recall that $x_0$ is the axial position of the wave vertex and $H_0$ is the radius of the undeformed tube at $x = x_0$). The second expression in Eq. (10) results from substituting the expression for $A$. When we apply this scaling factor to the peak centerline velocity $u_{\max}/U$, we get values of $(u_{\max}/U)\phi$ between 1.01 and 1.04 for the different wave geometries. This suggests that the peak centerline velocity in this computational geometry can be approximated by

$$u_{\max} = U\phi^{-1} = \frac{1}{(1 - RO)^2} \frac{2\sqrt{H_m H_0 \cos \theta}}{\lambda} |s| \lambda^{3/2}.$$

Scaling the volumetric flow rate by $\phi$ yields values of $Q$ between 204 mm$^3$/s and 211 mm$^3$/s. Both the scaled peak centerline velocity and the scaled volumetric flow rate vary by about 3% over the different wave geometries. The scaled centerline velocity curves are shown in Fig. 12c.

A scaling law for the retropulsive jet length $L$ at low Reynolds numbers can also be proposed. First, note that if the undeformed tube representing the antrum has a constant radius, and then, for $Re \ll 1$, it can be shown (using mass conservation) that

$$L = 2b \frac{\sqrt{H}}{A} = b\lambda \sqrt{\frac{1}{RO}} - 1.$$
for a parabolic-shaped wave, where \( b = \sqrt{2} - 1 = 0.6436 \). This relationship assumes that the half-peak velocity occurs within the width of the wave, i.e., \( L \leq \lambda \), and that the velocity profile in each cross section within \( L \) is parabolic. This former condition gives the restriction that \( RO \geq \frac{b^2}{1 + b^2} \approx 0.29 \). Under these idealized conditions, \( L/\lambda \) depends only on \( RO \), and not on \( \lambda \), and we find \( L/\lambda = 0.3218 \) for \( RO = 0.80 \) and \( L/\lambda = 0.6183 \) for \( RO = 0.52 \). Note that these idealized conditions will not exist in reality, even when the undeformed tube has a constant radius but should provide good approximations for wide waves where the geometry changes slowly, particularly near the bottom of the wave.

For the flow conditions considered here, \( L/\lambda \) is not independent of \( \lambda \). Figure 13(a) shows the curves of \( L/\lambda \) for the four wave geometries considered in our conical-shaped antrum. Note that the idealized values of \( L/\lambda = 0.3218 \) and 0.6183 come close to approximating \( L/\lambda \) for the wide waves. This figure also shows that \( L/\lambda \) is larger for the smaller relative occlusion and is influenced more by wave width for the smaller relative occlusion. Moreover, for small Reynolds numbers where the retropulsive jet is confined to the vicinity of the wave, the jet length is also influenced by the varying diameters over the width of the wave. If we let \( y_1 \) and \( y_2 \) be the radii of the tube at the tail end and leading end of the wave at maximum relative occlusion, respectively, then the ratio \( y_1/y_2 > 1 \) is a measure for the net change in diameter over the wave width. This ratio may be written as \( y_1/y_2 = (a + \lambda \sin \theta)/(a - \lambda \sin \theta) \), where \( a = 2H_0(1 - RO \sin^2 \theta) \) for a parabolic-shaped wave.

The above observations suggest a scaling of the \( L/\lambda \) curves by a factor involving the maximum relative occlusion and the diameter ratio \( y_1/y_2 \). Using the simple scaling factor of

\[
\psi = \left( \frac{y_1}{y_2} \right) RO, \quad (13)
\]

we see that the curves nearly coincide at low Reynolds numbers. This is shown in Fig. 13(b). The maximum difference in the scaled \( L/\lambda \) values at low Reynolds numbers is about 16% and occurs for the largest relative occlusion and the smallest wave width. The scaled jet length for this wave geometry is about 0.51 compared to 0.61 for the other three wave geometries. The reason for this may be due to the steepness of the tail portion of this wave (i.e., the part of the wave to the left of the vertex). Specifically, the axial position at which half the peak centerline velocity is obtained over the tail portion of this wave is nearly outside the width of the wave, and so the factor \( y_1/y_2 \) may under-represent the jet length. The values of the scaling factor \( \psi \) are given in Table II. Finally, note that in addition to the scaling at low Reynolds numbers, Fig. 13(b) also shows that for higher Reynolds numbers (up to \( Re \approx 40 \)), the scaled jet lengths are independent of relative occlusion.

As a further test of the above proposed scaling laws in Eqs. (10) and (13), additional simulations were performed at \( RO = 0.65 \) for a narrow wave (\( \lambda = 7 \) mm) and wide wave (\( \lambda = 18.5 \) mm). The wave speed was fixed to \( c = 2.3 \) mm/s and the fluid \( N_5 \) was used. We found that the scaled peak centerline velocity and the jet length agreed very well with the above scaled values. The scaled peak centerline velocity was 1.02 and 0.97 for the narrow and wide wave, respectively, while the scaled jet length was 0.61 and 0.60, respectively.

B. Recirculation and mixing characteristics

The second main flow pattern exhibited in ACW flow is the recirculation that occurs between and under the waves. The formation and strength of these regions of recirculation, or vortices, can be captured by streamlines and vorticity contours of the flow at the time that maximum relative occlusion is reached. The streamlines for the narrow wave at \( RO = 0.80 \) are shown in Fig. 3 for lowest and highest viscous fluids (\( N_1 \) and \( N_5 \), respectively) and the corresponding vorticity plots are shown in Fig. 14. Similar to the retropulsive jet, these figures illustrate that the formation and strength of vortices are sensitive to fluid viscosity. Specifically, the streamlines in Fig. 3 show that the low viscous fluid \( N_1 \) exhibits more of global recirculation, particularly close to the centerline, than the high viscous fluid \( N_5 \) and, hence, experiences enhanced global mixing. The high viscous fluid, on the other hand, exhibits more local recirculation near the waves. This characteristic is also seen in the vorticity plots in Fig. 14, where the vorticity contours of the high viscous fluid \( N_5 \) are confined to regions close to the ACWs. These results are consistent with experimental and numerical observations previously reported by Marciani et al.\(^6\) and Ferrua and Singh.\(^29\) They suggest that the gastric content associated with high viscous meals seems to be poorly mixed.

1. Vortex strength

The strength of the recirculation between consecutive antral contraction waves was quantified by the volume-averaged vorticity magnitude, \( |\omega_{avg}| \). This is illustrated in Fig. 14 for the narrow wave at \( RO = 0.80 \). For each fluid, we see that the vorticities strengthen
as the ACW propagates toward the pylorus and reach the maximum $|\omega_{\text{AVG}}|$ in the most occluded section of the stomach. Figure 15 shows that this is true for all four wave geometries. This figure shows $|\omega_{\text{AVG}}|$ as a function of the Reynolds number for all three regions depicted in Fig. 14. There are four sets of vertically positioned data points that, from left to right, correspond to (i) $N_2$, RO = 0.52, (ii) $N_3$, RO = 0.80, (iii) $N_4$, RO = 0.52, and (iv) $N_1$, RO = 0.80. We see that $|\omega_{\text{AVG}}|$ increases as the relative occlusion increases. Moreover, as the Reynolds number increases, each set of vertical points tends to spread out, that is, the vertical distances tend to increase. This can be seen by comparing, e.g., the vertical distances between the closed symbols (narrow wave). This indicates that for each wave width, the variation in $|\omega_{\text{AVG}}|$ over the different regions increases as the Reynolds number increases. The figure also shows that between the two most occluded ACWs (Region 1), $|\omega_{\text{AVG}}|$ is independent of wave width (as is indicated by the open and closed circles).

### 2. Particle tracking and mixing characteristics

Antral mixing was investigated by tracking particles (or fluid elements) in the flow field over a given period of time. This was done for a wave speed of $c = 2.3$ mm/s. The tracking ends when maximum relative occlusion of the front wave is reached, which corresponded to time periods of 60 s and 63 s for the wide and the narrow waves, respectively. The tracking process is illustrated in Fig. 16 for the narrow wave at RO = 0.80. The black numbered spheres represent the initial location of the particles, the white spheres represent their final location, and the white curves indicate their paths. Three sets of four particles each were positioned in the antrum. The first set of particles (spheres 1–4) was placed in a region near the pylorus where the front ACW will reach maximum relative occlusion at the end of the tracking time. This region is where the retropulsive jet is the strongest. The second set of particles (spheres 5–8) was placed in the region that will lie between the front two ACWs. This is the region of highest recirculation (i.e., region 1 in Fig. 14). The third set of particles (spheres 9–12) was placed near the antrum wall in a region that will lie in front of and behind the ACW farthest from the pylorus (to determine the particle paths near the antrum wall). Figure 16 shows that as the ACW propagates toward the pylorus, the fluid near the antrum wall is transported toward the pylorus (spheres 6, 7, and 9–12), whereas the material away from the wall is transported back into the corpus of the stomach (spheres 5, 8, and 1–4).

The strength of antral mixing was quantified using the mixing parameter proposed by Pal et al. Given a set $k$ of $N_k$ tracer particles, this parameter is defined as

$$M_k = \frac{R(t^n, k)}{R(t^n, 0)},$$

which gives the relative spread of the particles between time $t^n$ and $t^0$. The spread is measured by using the root mean square radius (or mixing radius) that is calculated by

$$R(t^n, k) = \sqrt{\frac{1}{N_k} \sum_{j=1}^{N_k} \left[ (x^n_j - x_m^n[k])^2 + (y^n_j - y_m^n[k])^2 + (z^n_j - z_m^n[k])^2 \right]},$$

or

$$R(t^n, k) = \frac{1}{N_k} \sum_{j=1}^{N_k} \left[ (x^n_j - x_m^n[k])^2 + (y^n_j - y_m^n[k])^2 + (z^n_j - z_m^n[k])^2 \right].$$
where \((x_n^j, y_n^j, z_n^j)\) is the position vector of particle \(j\) at time \(t_n^j\), and \((x_m^k[k], y_m^k[k], z_m^k[k])\) is the center of mass of the \(N_k\) particles in the set \(k\) at time \(t^k\). Since our particles are fluid elements, the mass is the same, and hence, the center of mass of set \(k\) is computed by averaging the position vector over the \(N_k\) particles. Note that \(M_k \geq 1\) and that a larger \(M_k\) corresponds to a wider spreading of the particles relative to their initial distribution. Therefore, a larger \(M_k\) is interpreted as better mixing.

Figure 17 shows the mixing parameter for fluids \(N_1\) (least viscous) and \(N_5\) (most viscous). The particle sets labeled Set 1 and Set 2 in Fig. 16 were considered since they are located in the region of the strongest retropulsive jet and strongest recirculation, respectively. The bar graph on the left shows the results for three of the wave geometries, and the graph on the right shows the results as a function of the Reynolds number for all four wave geometries. This figure shows that the mixing strength increases when the relative occlusion is increased or when the wave width is decreased. This is true for both fluids and particle sets. However, in the region between ACWs (Set 2), the wider wave at \(RO = 0.80\) creates more intensive mixing than the narrow wave at \(RO = 0.52\) for both fluids. Comparison of particle location shows that the low viscous fluid \(N_1\) is better mixed near the pylorus (Set 1) than in between ACWs (Set 2). The opposite is true for the high viscous fluid \(N_5\) at \(RO = 0.80\). While more intensive mixing occurs for the low viscous fluid \(N_1\) (data points on the right) than for the high viscous \(N_5\) (data points on the left) under a fixed set of conditions, the graph indicates that the mixing strength for particles in Set 2 varies nonmonotonically with respect to the Reynolds number. In particular, the mixing parameter for the high viscous fluid at \(RO = 0.80\) is larger than that for the low viscous fluid at \(RO = 0.52\). This suggests a complex interplay between the parameters governing the mixing characteristics.

IV. SUMMARY AND CONCLUSIONS

Numerical simulations were performed to obtain a better understanding of the flow fields that develop within the antrum during mixing at the beginning of the digestion process. A simplified computational model for the antrum was developed that allowed a parametric study to be performed efficiently. The computational model consisted of an axisymmetric cross section of a tube with the
linearly varying radius that was closed at the narrow end to represent the closed pylorus. The dimensions of the geometry were chosen to reflect those in an average sized human stomach. Peristaltic motion was induced by a sequence of parabolic-shaped ACWs traveling along the domain boundary. This computational model and boundary motion algorithm were implemented into the open source software package OpenFOAM, and the simulations were performed in the Eulerian frame of reference using dynamic meshing techniques. Four parameters were varied in the simulations: the kinematic viscosity $\nu$, the wave speed $c$, the maximum relative occlusion RO, and the wave width $\lambda$.

Results from our model were validated with those from a three-dimensional model of a larger portion of the stomach. We found that our model predicts nearly the same peak velocity and volume-averaged vorticity near the pylorus at maximum relative occlusion. More generally, comparison of our results with other numerical and experimental results in the literature established that our simplified model captures the essential ACW flow features. In particular, two basic antral flow patterns are formed as the ACWs propagate toward the pylorus: a backward retropulsive jet flow that is developed in the most highly occluded region near the pylorus and a forward recirculation flow that is developed between and under the ACWs. These flow patterns, both of which contribute to food disintegration and mixing, were studied in more detail. In general, the recirculation flow causes higher viscous fluids to experience less mixing than lower viscous ones, while the retropulsive jet causes these higher viscous fluids to experience higher strain rates (over a smaller region) and stresses, which helps in the breakdown of food. When the food breaks down, the viscosity decreases and the food then becomes better mixed in the stomach.

The flow in the retropulsive jet was characterized by the velocity and pressure along the centerline when maximum relative occlusion is reached. For $Re < 1$ (high viscous fluids), velocity and pressure curves are scaled with $c$ and $\nu$, respectively. For a fixed wave geometry (i.e., fixed RO and $\lambda$), this is equivalent to scaling the velocity with the characteristic velocity $U = c(H_0/H)^2$ and scaling the pressure with the characteristic pressure $P = \nu U/H = \nu c(H_0/H)^2/H$.

When the wave geometry was changed, the velocity and pressure curves did not scale so directly. Alternatively, we defined the strength of the retropulsive jet to be quantified by its peak centerline velocity under the leading ACW at maximum relative occlusion and its length at the half peak velocity. A parametric study then showed the effect of parameter variations on these quantities. It was found that, when $Re > 1$, the peak centerline velocity decreased and the length of the retropulsive jet increased when the fluid viscosity decreased, while these quantities were independent of fluid viscosity when $Re < 1$. Increasing the wave speed increased the peak centerline velocity by the same factor for $Re < 1$, but did not affect the jet length. For all Reynolds numbers considered, the peak centerline velocity increased and the jet length decreased when either the relative occlusion increased or the wave width decreased.

Based on the above observations and other physical considerations, scaling factors were proposed for the peak centerline velocity and the jet length in terms of the wave geometry. Applying this scaling to all four wave geometries produced the same jet lengths and peak centerline velocity for low Reynolds numbers.

The strength of the recirculation flow was quantified by the volume-averaged vorticity magnitude between pairs of consecutive ACWs when maximum relative occlusion is reached. Not surprisingly, this quantity was larger for low viscous fluids and larger near the pylorus. In the recirculation region closest to the pylorus, it was also found to be independent of wave width, while its variation over the different recirculation regions increased with Reynolds number. Finally, tracking sets of particles over time to quantify the mixing strength showed that mixing is enhanced in the most occluded region of the antrum when the relative occlusion increases, as well as when the viscosity or wave width decreases. As a consequence, gastric contents associated with high viscous meals are seen to be poorly mixed, while higher relative occlusion or narrower ACWs strengthen gastric fluid motions and enhance mixing.

Several issues in this study motivate further investigation, some of which are ongoing. First, the proposed scaling laws for different wave geometries should be further tested and possibly refined. This requires collecting simulation data from additional wave geometries. In this context, the effect of the inclination angle of the undeformed tube may also be of interest. Any proposed scaling principles should also be tested in more complex three-dimensional models of the stomach. Second, the relationship between the disintegration of food particles due to mechanical stresses and the history of strain rates experienced by fluid particles in ACW flow should be addressed. This history, and its decomposition into shear and extensional contributions, can be obtained using particle tracking techniques. Antral mixing and the mechanical disintegration of food particles may also be studied by determining the dispersing properties of ACW flow, that is, by determining breakup conditions of liquid drops with suitable material properties. For this, our computational model can be easily extended from a two-dimensional axisymmetric cross section to a three-dimensional wedge and still retain its relative computational efficiency. Finally, additional physics, such as chemical processes during digestion or non-Newtonian flow effects, can be efficiently studied with our computational model due to its relatively low computational cost.

REFERENCES


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