

# OTIS-Star an Attractive Alternative Network

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*Abstract:-* This paper proposes a new interconnection network referred to as the OTIS-Star which is constructed from multiplying a factor network star  $S_n$  by it self. In this paper we utilize the features of OTIS networks which use both of electronic and optical networks. Although the star graph has been an important research issue for long time it suffers from many problems mainly its limited efficiency in routing and broadcasting. The OTIS-Star presents a solution for the problems which star networks suffer from. In this paper we conduct a general study on the topological properties for the OTIS-Star by obtaining the main topological properties including size, degree, diameter and number of links, then we propose an efficient broadcasting algorithm.

*Key-words:-* Parallel and distributed systems; Interconnection Networks; Star network; OTIS-Networks; Topological properties; Broadcasting.

## 1 Introduction

Improving computational power is a decidable dimension in future technology trends. The solution to many real life problems is now possible due to the ability of high-speed parallel computers with hundreds or thousands of processors. In fact, a significant amount of the time required for a high-speed parallel computer to solve a given problem was spent by one or more processors communicating the results of their computations to other processors of the system. These processors are physically interconnected by an interconnection network, and co-ordinate their activities to solve a common problem by exchanging information, depending on the way communication is achieved between the processors.

A major design issue involved in the construction of high-speed parallel computers is the topology of the network interconnecting its processors. The power of high-speed parallel computers is mainly attributed to the efficiency of the topology interconnecting the processors. Among all the studied networks, none can be claimed to outperform all other networks [1,10,12,13]. Many networks have emerged as attractive interconnection networks for parallel computers. The most popular of these interconnection

networks are the hypercube [12], the torus [25], the star [1], and the arrangement network [13].

During the last decade a large variety of interconnection networks for parallel computers have been investigated [1, 10, 12, 13, 23]. One of the well-known interconnection networks is the star graph [1], which has been proposed as an attractive alternative to the hypercube [22]. Figure 1 shows the topology of this network. The star graph has attracted a lot of research efforts. For instance, several properties of this network have been studied including its basic topological properties [1], parallel paths characterization [14], and embedding [15]. Akers and Krishnamurthy [1] have shown that the star graph has several advantages over the hypercube including a smaller diameter, smaller average diameter and lower degree for a fixed network size. Also the star graph has edge and vertex symmetries that maximize its fault tolerant capabilities [1,16]. Furthermore, a number of parallel algorithms on the star network for some well-known problems have been reported in the literature including computing fast Fourier transforms and broadcasting [17].

The star graph, however, suffers from some drawbacks [18]. One major problem with the star graph is related to its limited efficiency in routing

and broadcasting. Despite its attractive topological properties, the star graph has not been used in practical systems yet. One reason for this may be attributed to the difficulty in developing parallel algorithms on this network for common parallel applications. Mapping of data and tasks on the star graph is not as obvious as it is the case with the hypercube or mesh [18]. To fill this gap we have proposed the OTIS-Star as an attractive alternative for the star network by utilizing both electronic and optical technologies.

A lot of research efforts have been devoted to Optical Transpose Interconnection Systems (OTIS) networks [4, 5, 7, 8]. This kind of networks is basically constructed by “multiplying” a factor topology by itself. It is implemented using both electronic interconnect technologies and free-space optical. In the last decade there was an increasing interest in the study of parallel algorithms for optoelectronic networks in general and for OTIS-networks in particular [4, 6, 8, 9, 11]. Sahni and Wang [9] have presented and evaluated a range of algorithms on OTIS-mesh (a special case of OTIS-networks), such as , routing, selection, basic data rearrangements and sorting. Furthermore Sahni and Wang have also developed algorithms for various matrix multiplication operations [19] and image processing [20].

Extensive modelling results for the OTIS have been reported in [5, 7]. The achievable Terra bit throughput at a reasonable cost makes the OTIS a strong competitor to the electronic alternatives [5, 8]. These encouraging findings prompt the need for further testing of the suitability of the OTIS for real-life applications in general and for OTIS-Star in specific. A number of recent studies have been conducted in this direction [3,4,6,19]. Zane et al [8] have shown that the OTIS-mesh efficiently embeds four-dimensional meshes and hypercubes.

In this paper, we focus on the Optical Transpose Interconnection Systems (OTIS) and its implementations by proposing OTIS-Star. OTIS-Star can be easily implemented using free-space optoelectronic technologies [6]. In this model, processors are partitioned into groups, where each group is realized on a separate chip with electronic inter-processor connects. Processors on separate chips are interconnected through free space interconnects. The philosophy behind this separation is to utilize the benefits of both the optical and the electronic technologies.

The advantage of using OTIS-Star as an optoelectronic architecture lies in its ability to deal with the fact that free space optical communication is superior in terms of speed and power consumption when the connection distance is more than few

millimetres [7]. In the OTIS-Star, shorter (intra-chip) communication is realized by electronic interconnects while longer (inter-chip) communication is realized by free space interconnects.

Since the study of proposing efficient OTIS networks still not been investigated widely we contribute towards filling this gap by proposing the OTIS-Star and conducting a general study on the topological properties including size, degree, diameter, shortest distance and number of links beside proposing algorithm for broadcasting of OTIS-Star network. Figure 2 shows the topology of OTIS-Star network.

This paper considers the “multiplication” of the star graph by itself with the aim to enhance the topological characteristics of the factor star network [21]. As we shall see below, our study has derived the topological properties of the OTIS-Star and it will enables a solution for the problems, which the factor network suffers from. The OTIS-Star network also preserves all the nice qualities of the star graph topologies including, hierarchical structure, simple shortest path routing and many other properties.

The rest of the paper is organized as follows . Section 2 provides the necessary notations and definitions, and then formally presents the OTIS-Star. Section 3 discusses some of the general topological properties of the OTIS-Star network. Section 4 proposes an efficient broadcasting algorithm for the OTIS-Star. Finally section 5 concludes this study.

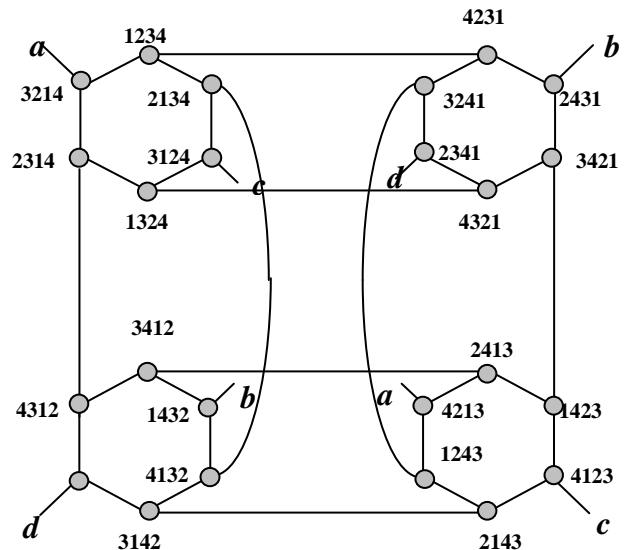


Fig.1: The 4-star graph

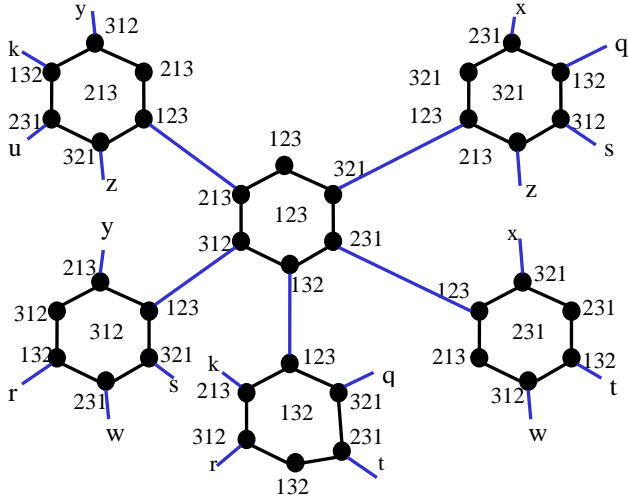


Fig.2: OTIS-Star network

## 2 Notations and definitions

**Definition 1:** The  $n$ -star graph, denoted by  $S_n$ , has  $n!$  nodes each labelled with a unique permutation on  $\langle n \rangle = \{1, \dots, n\}$ . Any two nodes are connected if, and only if, their corresponding permutations differ exactly in the first and one other position.

The diameter,  $\delta$ , and the degree,  $\alpha$ , of the star graph are as follows [3]:

$$\delta, \text{ of } n\text{-star graph} = \lfloor 1.5(n-1) \rfloor$$

$$\alpha, \text{ of the } n\text{-star graph} = n-1, \text{ where } n > 1.$$

An OTIS based computer contains  $N^2$  processors partitioned into  $N$  groups with  $N$  processors each. A processor is indexed by a pair  $\langle x, y \rangle$ ,  $0 \leq x, y < N$  where  $x$  is the group index and  $y$  is the processor index. Processors within the same group are connected by a certain interconnecting topology; while inter-group links are achieved by transposing group and processor indexes. The terms OTIS-computer and OTIS-networks refer to parallel architectures based on the OTIS and will be used interchangeably in the rest of the paper.

The OTIS-network is constructed by "multiplying" a known topology by itself. The vertex set is equal to the Cartesian product on the vertex set in the factor network. The edge set consists of edges from the factor network and new edges called the *transpose* edges. The formal definition of the OTIS-network is given below.

**Definition 2:** Let  $G_0 = (V_0, E_0)$  be an undirected graph representing a factor network. The OTIS- $G_0 = (V, E)$  network is represented by an undirected graph obtained from  $G_0$  as follows  $V = \{\langle x, y \rangle \mid x, y \in V_0 \text{ such that } x \neq y\}$  and  $E = \{\langle \langle x, y \rangle, \langle x, z \rangle \rangle \mid \text{if } (y, z) \in E_0\} \cup \{\langle \langle x, y \rangle, \langle y, x \rangle \rangle \mid x, y \in V_0 \text{ such that } x \neq y\}$ .

From the above definition the set of edges  $E$  consists of two subsets, one is from  $G_0$ , called  $G_0$ -type edges, and the other subset contains the transpose edges. The OTIS approach suggests implementing  $G_0$ -type edges by electronic links since they involve intra-chip short links and implementing transpose edges by free space optics. Throughout this paper, the terms "electronic move" and the "OTIS move" (or "optical move") will be used to refer to data transmission based on electronic and optical technologies, respectively.

**Definition 3:** Let  $S_n = (V_0, E_0)$  be an undirected star graph representing a factor network. The OTIS- $S_n = (V, E)$  network is represented by an undirected graph obtained from  $S_n$  as follows  $V = \{\langle x, y \rangle \mid x, y \in V_0 \text{ such that } x \neq y\}$  and  $E = \{\langle \langle x, y \rangle, \langle x, z \rangle \rangle \mid \text{if } (y, z) \in E_0\} \cup \{\langle \langle x, y \rangle, \langle y, x \rangle \rangle \mid x, y \in V_0 \text{ such that } x \neq y\}$ .

## 3 General Topological properties

This section discusses some of the basic topological properties of the OTIS-Star network including size, degree, shortest distance between 2 nodes, diameter, and number of links. The topological properties of the OTIS-Star network along with those of the star network are discussed and derived below. These topological properties have been derived using the theoretical framework for analyzing topological properties of the OTIS-Networks in general which was proposed by Al-Ayyoub and Day [2], beside other related research work [9].

In this paper we will refer to  $g$  as the group address and  $p$  as the processor address. An intergroup edge of the form  $\langle \langle g, p \rangle, \langle p, g \rangle \rangle$  represents an optical link and will be referred to as OTIS or optical move. Note that also we will be using the following notations through our paper:

- $|S_n|$  = size of the graph  $S_n$ .
- $|\text{OTIS-}S_n|$  = size of the graph OTIS- $S_n$ .
- $\text{Deg. } S_n(p)$  = Degree of the graph  $S_n$  at node  $p$ .
- $\text{Deg. OTIS-}S_n(g,p)$  = Degree of the graph OTIS- $S_n$  at node address  $\langle g, p \rangle$ .

- $\text{Dist}_{S_n}(p_1, p_2)$  = The length of a shortest path between the two nodes  $p_1$  and  $p_2$  in Star graph as indicated below.
- $\text{Dist. OTIS-}S_n(p_1, p_2)$  = The length of a shortest path between the two nodes  $\langle g_1, p_1 \rangle$  and  $\langle g_2, p_2 \rangle$  in OTIS-Star.

In the OTIS-Star the notation  $\langle g, p \rangle$  is used to refer to the group and processor addresses, respectively. Two nodes  $\langle g_1, p_1 \rangle$  and  $\langle g_2, p_2 \rangle$  are connected if, and only if,  $g_1 = g_2$  and  $(p_1, p_2) \in E_0$  (such that  $E_0$  is the set of edges in star network) or  $g_1 = p_2$  and  $p_1 = g_2$ , in this case the two nodes are connected by transpose edge.

The distance in the OTIS-Star is defined as the shortest path between any two processors,  $\langle g_1, p_1 \rangle$  and  $\langle g_2, p_2 \rangle$ , and involves one of the following forms [24]:

- (1) When  $g_1 = g_2$  then the path involves only electronic moves from source node to destination node.
- (2) When  $g_1 \neq g_2$  and if the number of optical moves is an even number of moves and more than two, then the paths can be compressed into a shorter path of the form:  $\langle g_1, p_1 \rangle \xrightarrow{E} \langle g_1, p_2 \rangle \xrightarrow{O} \langle p_2, g_1 \rangle \xrightarrow{E} \langle p_2, g_2 \rangle \xrightarrow{O} \langle g_2, p_2 \rangle$  here the symbols O and E stand for optical and electronic moves respectively.
- (3) When  $g_1 \neq g_2$ , and the path involves an odd number of OTIS moves. In this case the paths can be compressed into a shorter path of the form:

$$\langle g_1, p_1 \rangle \xrightarrow{E} \langle g_1, g_2 \rangle \xrightarrow{O} \langle g_2, g_1 \rangle \xrightarrow{E} \langle g_2, p_2 \rangle.$$

The following are the basic topological properties for the OTIS-Star. For Instance if the factor star network is of size  $n!$ , degree is  $n-1$  and diameter is  $\lfloor 1.5(n-1) \rfloor$ . Then the size, the degree, the diameter, number of links, and the shortest distance of OTIS-Star network are as follows:

- Size of  $|\text{OTIS-}S_n| = |n!|^2$ .
- Degree of  $\text{OTIS-}S_n = \text{Deg.}(S_n)$ , if  $g = p$ .
- $\text{Deg.}(S_n) + 1$ , if  $g \neq p$ .
- Diameter of  $\text{OTIS-}S_n = 2\lfloor 1.5(n-1) \rfloor + 1$ .

Number of Links: Let  $N_0$  be the number of links in the  $S_n$  and let  $M$  be the number of nodes in the  $\text{OTIS-}S_n$ . The number of links in the  $\text{OTIS-}S_n = (M^2 - M) / 2 + N_0^2$ . For instance, the number of links in the  $\text{OTIS-}S_3$  consisting of 6 processors is = 40.

- $\text{Dist. of OTIS-}S_n = \begin{cases} \min(d(p_1, g_2) + d(g_1, p_2) + 1, d(p_1, p_2) + d(g_1, g_2) + 2) & \text{if } g_1 \neq g_2 \\ \text{Dist.}(p_1, p_2) & \text{if } g_1 = g_2 \end{cases}$

## 4 Broadcasting in OTIS-Star Network

One-to-all broadcasting is a communication process in which a single source node sends the same message to every other node in the network. It is used in many applications. Examples are Gauss elimination method, and matrix-vector multiplication [26]. An optimal broadcasting algorithm for single communication port star networks is presented in [27]. This algorithm takes  $(n-1)(\lg(n-1)+1)$  time steps in  $S_n$ . For multi-communication ports star networks a broadcasting algorithm that takes  $(2n-3)$  time step is introduced in [28].

Broadcasting in  $\text{OTIS-}S_n$  is based on broadcasting in  $S_n$  networks as follows:

*Step1:  $\langle g_i, s_j \rangle$  broadcasts  $M$  locally to every node in the group  $\langle g_i \rangle$ , where  $\langle g_i, s_j \rangle$  is the source node and  $M$  is the message to be broadcasted.*

*Step 2: Every node  $\langle g_i, s_k \rangle$ ,  $1 \leq k \leq n!, k \neq i$ , sends  $M$  to  $\langle s_k, g_i \rangle$  over the optical link.*

*Step 3: Every node  $\langle s_k, g_i \rangle$ ,  $1 \leq k \leq n!, k \neq i$ , broadcasts  $M$  locally in the  $\langle s_k \rangle$  group.*

Using the algorithms in [27] and [28] for local broadcasting in the group star networks, the above algorithm takes  $2(n-1)(\lg(n-1)+1)+1$  steps, or  $(4n-5)$  steps in an  $\text{OTIS-}S_n$  network depending whether it is single or multi-communication port, respectively.

## 5 Conclusion

In this paper we have contributed to the study of OTIS-Networks in general and to OTIS-Star in particular through proposing the OTIS-Star network as an attractive network to its factor network. Several topological properties including size, degree, diameter, number of links and shortest distance between any two nodes have been derived. Efficient broadcasting algorithm is introduced for the OTIS-Star. The proposed OTIS-Star could be an attractive alternative over its factor network. It has more efficient broadcasting algorithm compared to typical

star networks. Furthermore, OTIS-Star can take advantage of deploying both electronic and optical technologies. As a future research work we could utilize the proposed OTIS-Star in solving real life problems on OTIS-Star including Matrix problems, Fast Fourier transforms. Furthermore proposing an efficient routing for OTIS-Star will be also one of our targets as a future research.

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